Fiscal-Monetary Interactions and the FTPL:
Coordination and Value of Information

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Morris and Shin (2002)

Social Value of Public Information

- Paradigm: more info always better for policymaker under uncertainty, regardless of "type" of info (public or private)
- Big but: public info has 2 roles:
  - reveal info on objects of interest
  - serves as focal point for beliefs of private sector ($\downarrow$ strategic uncertainty)
- This paper: study value of public info allowing for these 2 roles

Main Results

- "Beauty contest" model: agents value both doing the right thing *and* doing what the others do
- With perfect info: unique eqm, first best
- With imperfect info:
  - if only public info, welfare $\uparrow$ in info precision
  - if public $+$ private, public info has ambiguous effects
- Intuition: coordination motive makes agent overweight public info
- Several implications for policy communication: frequency vs precision of information

- Continuum of agents $i \in [0,1]$, action $a_i \in \mathbb{R}$
- Payoff

$$u_i(\mathbf{a}, \theta) := -(1-r)(a_i - \theta)^2 - r(L_i - \bar{L})$$

where the beauty contest term is given by

$$L_i := \int_0^1 (a_j - a_i)^2 \mathrm{d}j, \quad \bar{L} := \int_0^1 L_j \mathrm{d}j$$

can rewrite it as

$$u_i(\mathbf{a}, \theta) := -(1-r)(a_i - \theta)^2 - r(\bar{a} - a_i)^2 + r\sigma_a^2$$

- Agent i maximises expected utility, plays

$$a_i = (1-r)\mathbb{E}_i(\theta) + r\mathbb{E}_i(\bar{a})$$

- Social welfare

$$W(\mathbf{a}, \theta) = \frac{1}{1-r} \int_0^1 u_i(\mathbf{a}, \theta)\mathrm{d}i = -\int_0^1 (a_i - \theta)^2 \mathrm{d}i$$

$\Rightarrow$ coordination motive externality, strength $r \in (0,1)$, zero-sum game

Public Info Only

Common knowledge

- If θ is common knowledge, $a_i = \theta \, \forall i$, social welfare is maximised

Noisy public info

- Assume a noisy *public* signal

$$y = \theta + \eta, \quad \eta \sim N(0, \sigma_\eta^2)$$

- All agents have identical beliefs $\theta \mid y \sim N(y, \sigma_\eta^2)$, choose action

$$a_i(y) = (1 - r)\mathbb{E}[\theta \mid y] + r \int_0^1 \mathbb{E}[a_j \mid y] \mathrm{d}j$$

- Taking conditional expectations, we get $\mathbb{E}[a_i(y) \mid y] = \mathbb{E}[\theta \mid y] = y$
- In the unique symmetric eqm $a_i(y) = y$
- Welfare $\downarrow$ in σ_η

$$\mathbb{E}[W \mid \theta] = -\mathbb{E}[(y - \theta)^2 \mid \theta] = -\sigma_\eta^2$$

Private + Public Info

- Additional private signal iid across agents

$$x_i = \theta + \epsilon_i, \quad \epsilon_i \sim N(0, \sigma_\epsilon^2)$$

- Agent's actions are now $a_i(y, x_i)$ ($\neq$ info sets across agents)

- Let $\alpha := 1/\sigma_\eta^2, \beta := 1/\sigma_\epsilon^2$ denote signal precisions (inverse of variance)

- Agent i's posterior beliefs are given by

$$\theta \,|\, y, x_i \sim N\left(\frac{\alpha y + \beta x_i}{\alpha + \beta}, \frac{1}{\alpha + \beta}\right)$$

$\Rightarrow$ signal with higher precision receives higher weight

Linear Equilibrium

- Conjecture linear strategies

$$a_i(y, x_i) = \kappa x_i + (1-\kappa)y$$

- Then the expected average action is

$$\mathbb{E}_i(\bar{a}) = \kappa \frac{\alpha y + \beta x_i}{\alpha + \beta} + (1-\kappa)y$$

- Agent i's optimal action is

$$a_i(y, x_i) = (1-r)\mathbb{E}_i(\theta) + r\mathbb{E}_i[\bar{a}] \;=\; \frac{\beta(r\kappa + 1 - r)}{\alpha + \beta} x_i + \left(1 - \frac{\beta(r\kappa + 1 - r)}{\alpha + \beta}\right) y$$

- Solving for κ

$$a_i(y, x_i) = \frac{\alpha y + \beta(1-r)x_i}{\alpha + \beta(1-r)}$$

Equilibrium Properties

$$a_i(y, x_i) = \frac{\alpha y + \beta(1 - r)x_i}{\alpha + \beta(1 - r)}$$

- With no coordination motive ($r = 0$), standard Bayesian updating signal extraction problem
- Higher $r \Rightarrow$ higher weight attached to public signal

For uniqueness, work with higher order beliefs (forecasting others' forecasts)

$$\begin{aligned}
a_i &= (1 - r)\mathbb{E}_i(\theta) + r\mathbb{E}_i[\bar{a}] \\
\bar{a} &= (1 - r)\bar{\mathbb{E}}(\theta) + r\bar{\mathbb{E}}[\bar{a}] \\
&= (1 - r)\bar{\mathbb{E}}(\theta) + r\bar{\mathbb{E}}\Big[(1 - r)\bar{\mathbb{E}}(\theta) + r\bar{\mathbb{E}}[\bar{a}]\Big] \\
&= (1 - r)\sum_{k=0}^{\infty} r^k \bar{\mathbb{E}}^k(\theta)
\end{aligned}$$

Uniqueness

- To evaluate expression $(1-r)\sum_{k=0}^{\infty} r^k \bar{\mathbb{E}}^k(\theta)$, algebra shows that

$$\bar{\mathbb{E}}^k(\theta) = (1-\mu^k)y + \mu^k\theta$$
$$\mathbb{E}_i[\bar{\mathbb{E}}^k(\theta)] = (1-\mu^{k+1})y + \mu^{k+1}x_i$$

  with $\mu = \frac{\beta}{\alpha+\beta}$
- Weight on public info $\uparrow$ with k: y observed by everyone, more useful to forecast what others know
- Plugging into a_i, we obtain

$$a_i(y, x_i) = \frac{\alpha y + \beta(1-r)x_i}{\alpha + \beta(1-r)}$$

  which implies the linear eqm found earlier is unique

One Interpretation

Ok, but what exactly is r? Lucas-Phelps island model from Myatt and Wallace (2014)

- Continuum of islands, each denoted by i, aggregate demand drive by θ
- In each, producers choose quantities to be sold at a price p_i
- Supply and demand (gaps) are given by

$$y_i^s = b[p_i - \mathbb{E}_i(\bar{p})]$$
$$y_i^d = c[\mathbb{E}_i(\theta) - p_i]$$

- Market clearing implies

$$p_i = (1-r)\mathbb{E}_i(\theta) + r\mathbb{E}_i(\bar{p}), \quad \text{with} \quad r = \frac{b}{b+c}$$

  coordination motive = relative size of demand vs supply elasticity
- Public signal precision = optimal communication policy!
- Same structure arises with Bertrand competition and differentiated suppliers, investment games with complementarities, Cournot games

Welfare

- Main question: how does welfare depends on info precision α and β?
- Write agents' policy a_i as a function of θ, η, ϵ_i

$$a_i = \theta + \frac{\alpha\eta + \beta(1-r)\epsilon_i}{\alpha + \beta(1-r)}$$

- If $\neq 0$, noise weights $\neq$ their precision
  - if $r > 0$, bias towards public info (desire to coordinate)
  - if $r < 0$, bias towards private info (desire to differentiate)
- Welfare

$$\mathbb{E}[W \mid \theta] = -\int_0^1 (a_i - \theta)^2 \mathrm{d}i = -\frac{\alpha + \beta(1-r)^2}{[\alpha + \beta(1-r)]^2}$$

Comparative Statics

Private information

Welfare

$$\mathbb{E}[W \mid \theta] = -\int_0^1 (a_i - \theta)^2 \mathrm{d}i = -\frac{\alpha + \beta(1-r)^2}{[\alpha + \beta(1-r)]^2}$$

Comparative statics

$$\frac{\partial \mathbb{E}(W \mid \theta)}{\partial \beta} = \frac{(1-r)[(1+r)\alpha + (1-r)^2\beta]}{[\alpha + \beta(1-r)]^3} > 0$$

Private info is always beneficial

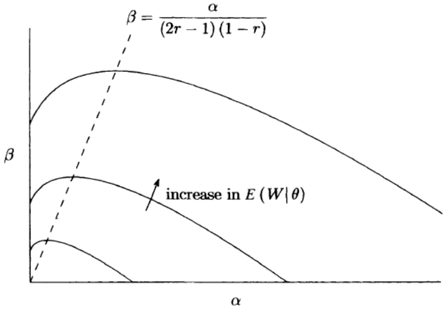

FIGURE 1. SOCIAL WELFARE CONTOURS

Comparative Statics

Public information

Welfare

$$\mathbb{E}[W \mid \theta] = -\int_0^1 (a_i - \theta)^2 \mathrm{d}i = -\frac{\alpha + \beta(1-r)^2}{[\alpha + \beta(1-r)]^2}$$

Comparative statics

$$\frac{\partial \mathbb{E}(W \mid \theta)}{\partial \alpha} = \frac{\alpha - (2r-1)(1-r)\beta}{[\alpha + \beta(1-r)]^3} \geq 0$$

$$\Leftrightarrow \quad \frac{\alpha}{\beta} \geq (2r-1)(1-r)$$

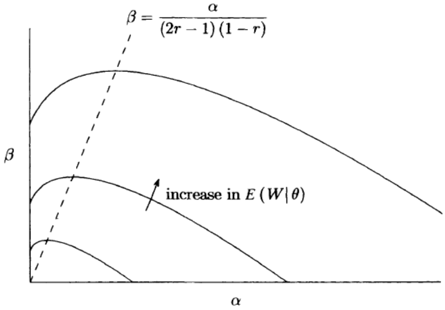

FIGURE 1. SOCIAL WELFARE CONTOURS

If $r > 1/2$, public info precision can be detrimental to welfare

e.g. if there is an upper bound $\bar{\alpha}$, then optimal precision is either 0 or $\bar{\alpha}$ depending on β

Zero transparency ($\alpha = 0$) dominates any $\alpha < \alpha^* := \beta(2r-1)$ (which is $\uparrow \beta$)

Intuition

- Can rewrite optimal actions as

$$a_i = \frac{\alpha y + \beta x_i}{\alpha + \beta} + (y - x_i)\left(\frac{\alpha}{\alpha + \beta}\right)\frac{\beta r}{\alpha + \beta(1 - r)}$$

  unbiased posterior + overreaction to public/underreaction to private info (if $r > 0$)

- Law of Iterated Expectations does *not* hold for the average expectation operator $\bar{\mathbb{E}}$

$$\bar{\mathbb{E}}(\theta) \neq \bar{\mathbb{E}}[\bar{\mathbb{E}}(\theta)] \quad \text{and} \quad \mathbb{E}_i(\theta) \neq \mathbb{E}_i[\bar{\mathbb{E}}(\theta)]$$

  if it did hold, then we would get the socially efficient solution

$$a_i = (1 - r)\sum_{k=0}^{\infty} r^k \mathbb{E}_i[\bar{\mathbb{E}}^k(\theta)] = \mathbb{E}_i[\bar{\mathbb{E}}(\theta)] = \mathbb{E}_i(\theta)$$

Extension

- Suppose N players and welfare depends on deviation of "aggregate" action relative to θ

$$W(\bar{a}, \theta) = (\bar{a} - \theta)^2 \qquad \text{where} \qquad \bar{a} := \frac{1}{n} \sum_i a_i$$

- The optimal Bayesian weights maximise

$$\begin{aligned}
\mathbb{E}[W(\bar{a}, \theta)] &= \mathbb{E}\left(\frac{1}{n} \sum \kappa(\theta + \epsilon_i) + (1 - \kappa)(\theta + \eta) - \theta\right)^2 \\
&= \mathbb{E}\left(\frac{\kappa}{n} \sum_i \epsilon_i + (1 - \kappa) y\right)^2 \\
&= \frac{\kappa^2}{n\beta} + \frac{(1 - \kappa)^2}{\alpha}
\end{aligned}$$

and $\kappa^* = \frac{n\beta}{\alpha + n\beta}$, which $\rightarrow 1$ as $n \rightarrow \infty$

Conclusion

- With complementarities, public info generates externalities
- More public info may be bad, generating undesired “herd” behaviour

References

Morris, Stephen and Hyun Song Shin, "Social Value of Public Information," *American Economic Review*, 2002, *92* (5), 1521–1534.

Myatt, David P. and Chris Wallace, "Central bank communication design in a Lucas-Phelps economy," *Journal of Monetary Economics*, 2014, *63*, 64–79.